Prompt Time Constants of a Reflected Reactor

*Tao Ye   Chaobin Chen   Weili Sun   Benai Zhang   Dongfeng Tian

Institute of Applied Physics and Computational Mathematics, Beijing, 100088, China
E-mail: yetao@aees.kyushu-u.ac.jp

Based on G. D. Spriggs' two-region kinetics model, a two-group point reactor kinetics model is developed. With the help of MCNP code, the modified model calculates prompt time decay constants of one benchmark reactor, PU-MET-FAST-024. The results of fundamental and secondary modes agree well with MCNP time fitting results in different subcritical reactivities.

1. INTRODUCTION

Time eigenvalue of transportation equation, alpha, is defined to describe all neutrons' time behavior (increasing or decreasing) in a nuclear reactor. Its number reflects the criticality also. The time constant, especially prompt time constant, had been studied for 60 years. Lots of reflected reactor's experimental data *cannot be satisfactorily explained using the standard point kinetic model* [1]. And multiple decay modes near delayed critical were also observed, which of course cannot be described by standard point kinetic model.

The existing numerical transportation codes, such as MCNP4C [2] and TART [3,4] , can do the job well with only the fundamental mode calculated. By using alpha static criticality method, MCNP4C is a good tool if k_{eff} is close 1, which means the reactor is near delayed critical. But MCNP4C's calculation may be very difficult and time-consuming if the reactor has more negative reactivity or reflector contains hydrogen, or both.

In the region of analytical method, many works contains too much mathematics, which are not easy to calculate and compare with experimental data. G. D. Spriggs' one-group, two-region kinetic model based on Avery-Cohn model is simple, calculable. The model introduces *simple probability relationships essential to calculating the coupling parameters between core and reflector*,[1] and derives the reflected-core inhour equation which contains multiple decay modes. However, Spriggs model cannot well describe multiple time constants of the thermal reflected reactor. In this kind of reactor, thermal neutrons with long lifetime contribute much to the time constant. Because of importance of thermal neutrons in such fast-thermal reactor, we present a simplified two-group, two-region kinetic model (2G2R) based on Spriggs model, and rewrite the reflected-core inhour equation. With the help of MCNP code, we calculated the coupling parameters, neutron lifetimes and first and secondary time constant of a spherical benchmark reactor, PU-MET-FAST-024.[6] Because we don't have experimental data, the results of time constants are also compare with 3 different models, MCNP time fitting method, alpha static method (MCNP4C), and Spriggs model. The results of 2G2R model agree well with MCNP time fitting method which can be thought as an experiment in computer.

2. ALPHA STATIC CRITICALITY METHOD

MCNP4C code introduced a new feature to calculate the fundamental mode of prompt time eigenvalue[2]. It is based on alpha static criticality method. In subcritical condition, the equation is

$$\upsilon\Omega\cdot\nabla N + \left(\sigma_t + \frac{|\alpha|}{\upsilon}\right)\cdot\upsilon N = \iint d\Omega' dE'\left(\sigma_s'\upsilon'N' + 2\frac{|\alpha|}{\upsilon}\cdot\upsilon'N'\delta(\Omega'-\Omega)\delta(E'-E)\right) \\ + \frac{1}{k'}\iint d\Omega' dE'\left(v\sigma_f\upsilon'N'\right) \tag{1}$$

The calculation procedures are to get k'≈1 by searching proper alpha. Then equation becomes

$$\upsilon\Omega\cdot\nabla N + \left(\sigma_t + \frac{|\alpha|}{\upsilon}\right)\cdot\upsilon N = \iint d\Omega' dE'\left(\sigma_s'\upsilon'N' + 2\frac{|\alpha|}{\upsilon}\cdot\upsilon'N'\delta(\Omega'-\Omega)\delta(E'-E)\right) \\ + \iint d\Omega' dE'\left(v\sigma_f\upsilon'N'\right) \tag{1'}$$

which is the alpha eigenequation.

If a reactor has more negative reactivity or reflector contains hydrogen, or both, the ratio between (α/v) term and σ_t term can be very large, which will results non-physical high particle weight and stops the calculation. We add an adjusting parameter to lower the ratio. The modified equation is

$$\upsilon\Omega\cdot\nabla N + \left(\sigma_t + \eta\frac{|\alpha|}{\upsilon}\right)\cdot\upsilon N = \iint d\Omega' dE'\left(\sigma_s'\upsilon'N' + (1+\eta)\frac{|\alpha|}{\upsilon}\cdot\upsilon'N'\delta(\Omega'-\Omega)\delta(E'-E)\right) \\ + \frac{1}{k'}\iint d\Omega' dE'\left(v\sigma_f\upsilon'N'\right) \tag{2}$$

The modification can only weaken the tendency of getting huge particle weight, and restrictedly extend the usage of MCNP4C. The *k'* intends to converge to a number larger than unit if reactor is in a deeper subcriticality, which means calculated alpha is smaller than true value in number axis. The determination of adjusting parameter is a little arbitrary based on various calculation conditions. Once confirmed, it shall not change in the running.

3. MCNP TIME FITTING

In time dependant transportation equation, neutron density or flux has the formal solution,[5]

$$N\left(\vec{r},\vec{\Omega},E,t\right) = \sum_{j=0} N_j\left(\vec{r},\vec{\Omega},E\right)\times e^{\alpha_j t}. \tag{3}$$

In a subcritical system, all α_j values are negative. We assume α_j's absolute values increase with increasing *j*. And α_0 is the largest one, the fundamental time constant. If we add a pulse source at zero time, the neutron's time distribution will start a buildup in the beginning, then drop to multiple decay mode which has nothing to do with source anymore.

By integrating volume, solid angle and energy, the current term becomes leakage term. But its time behavior still follows formula (3). which means we can use MCNP's tally option, F1, to count leakage neutrons' time distribution as system's time distribution. Then, we use formula (3) to fit time distribution to get multiple time constants. The fitting coefficients are time-independent flux corresponding to each decay mode. However, the coefficient, N_j, is not concerned in this work.

With enough neutron source particles (NPS) and adequate time, MCNP F1 tally can explain Rossi-α measurement well in the vicinity of delayed critical. At this point, MCNP F1 tally is doing same thing

as Rossi-α does. The difference is Rossi-α has background term, which is not easy to get rid of from experimental counts, and will conceal the lower decay modes, especially α_0, in a deeper subcritical system with a thermal reflector. To the contrary, MCNP F1 tally can display all lower decay modes without interference of background term. So, MCNP F1 tally can be seen as an imaginary Rossi-α measurement in computer.

4. 2G2R MODEL

We adopt the conventional diffusion approximation to deal with a two-region system consisting of a core surrounded by a non-multiplying, source-free reflector.[1] The simplified model can be described as the following set of two-group coupled differential equations.

$$
\begin{pmatrix} \dfrac{d\phi_{1c}}{\upsilon_{1c}dt} \\[2mm] \dfrac{d\phi_{2c}}{\upsilon_{2c}dt} \\[2mm] \dfrac{d\phi_{1r}}{\upsilon_{1r}dt} \\[2mm] \dfrac{d\phi_{2r}}{\upsilon_{2r}dt} \end{pmatrix} = \begin{pmatrix} -B_c^2 D_{1c} + (1-\beta_{eff})\nu\Sigma_{f_{1c}} - \Sigma_{r_{1c}} & (1-\beta_{eff})\nu\Sigma_{f_{2c}} & B_r^2 D_{1r} f_{rc_{11}} & 0 \\ \Sigma_{s_{12c}} & -B_c^2 D_{2c} - \Sigma_{a_{2c}} & 0 & B_r^2 D_{2r} f_{rc_{22}} \\ B_c^2 D_{1c} f_{cr_{11}} & 0 & -B_r^2 D_{1r} - \Sigma_{r_{1r}} & 0 \\ 0 & B_c^2 D_{2c} f_{cr_{22}} & \Sigma_{s_{12r}} & -B_r^2 D_{2r} - \Sigma_{a_{2r}} \end{pmatrix} \begin{pmatrix} \phi_{1c} \\ \phi_{2c} \\ \phi_{1r} \\ \phi_{2r} \end{pmatrix}
$$

(4)

Where subscript 1 represents the fast group ($E>1eV$), and 2 represents thermal group ($E<1eV$). c means core, and r means reflector. For simplification, we only include effective fraction of delayed neutrons, β_{eff}, in set of equations.

After Laplace transformation, we can get inhour equation,

$$
\left[(\omega l_{1c} + 1 - k_{1c}(1-\beta_{eff}))(\omega l_{1r}+1) - f_{11} \right] \times \left[(\omega l_{2c}+1)(\omega l_{2r}+1) - f_{22} \right]
$$
$$
- k_{2c}(1-\beta_{eff})\left[(\omega l_{1r}+1)(\omega l_{2r}+1)k_{s12c} + f_{12} \right] = 0
$$

(5)

In most reflected systems, the thermal neutron's number is a few orders smaller than fast neutron's number, which leads to a neglectable $f_{22}(\approx 0)$. And average thermal neutron's lifetime is sufficiently small such that $\omega_j l_{2c} \ll 1$ for all possible j roots. Introducing the definition of reactivity, the inhour equation can be rewritten as

$$
\omega\frac{l_{1c}}{k_c+\Delta} - \omega\frac{f_{11}l_{1r}}{(\omega l_{1r}+1)(k_c+\Delta)} - \frac{f_{12}\,k_{2c}(1-\beta_{eff})}{k_c+\Delta}\left[1 - \frac{1}{(\omega l_{1r}+1)(\omega l_{2r}+1)} \right] = 1-\beta_{eff} - \frac{1-f_{11}}{k_c+\Delta} = \rho - \beta_{eff}
$$

(6)

In many cases, f_{22} and k_{s12c} can be neglected and l_{2r} is very large such that $\omega_j l_{2r} \gg 1$, then the inhour equation is back to Spriggs model shape with one feedback constant, and only describe fast neutron's time constant. But we will lose a root related with l_{2r} by this simplification. It is better to resolve cubic equation

$$
\left[(\omega l_{1c}+1-k_{1c}(1-\beta_{eff}))(\omega l_{1r}+1) - f_{11} \right] \times (\omega l_{2r}+1) - k_{2c}(1-\beta_{eff})f_{12} = 0.
$$

(7)

with determined coefficients calculated by MCNP code.

5. MODEL CALCULATIONS

Because we don't have experimental data to test the 2G2R model, a benchmark model, PU-MET-FAST-024,[6] is chosen as preliminary test. A summary of the reactor is given in Table I. And 3 different models do the same calculations for comparison between them. One model, MCNP time fitting, is regarded as imaginary experiment because of similarity with Rossi-α measurement. All results are listed below.

Table I: Simplified ***PUT-MET-FAST-024*** Reactor Description [6]

Region/Dimension	Material	Atom Density($\times10^{-24}$) cm^{-3}
Core (spherical) 6*cm* radius	^{239}Pu	3.6620×10^{-2}
	^{240}Pu	6.6944×10^{-4}
	Ga	2.1962×10^{-3}
	Fe	1.4126×10^{4}
	O	2.8972×10^{-4}
	Ni	1.9748×10^{-3}
Reflector (spherical shell) 1.55*cm* thick	C	3.8814×10^{-2}
	H	7.7616×10^{-2}
	D	1.1644×10^{-5}

Table II: Integral Quantities with different reflector thick

| Model / Reflector's thick	0cm Bare Reactor	0.6cm	1.0cm	1.55cm
k_{eff}				
MCNP	0.92311	0.95323	0.97277	0.99823
Fundamental Time Constant, α_0 (μs^{-1})				
Alpha Static Method	-25.88	-2.372	-0.45	-0.035
MCNP Time Fitting	-29.48	-1.23	-0.53	-0.038
Spriggs Model[a]	-22.43	-10.0	-3.68	-0.16
2G2R Model[a]	-22.40	-0.57	-0.26	-0.20
Secondary Time Constant, α_1 (μs^{-1})				
MCNP Time Fitting	—	-8.26	-5.10	-0.37
2G2R Model	—	-10.21	-4.55	-0.82

a－β_{eff}=0.0023 is taken.

Figures a to d display the MCNP time fitting results. Fig. a is for bare system, and Fig. d is for critical system. In Fig. b and c, neutron counts shows a sharp drop of 2 orders in a few microseconds and still do not enter the fundamental decay mode. Rossi-α measurement will be difficult to get the fundamental mode for such reactors, because secondary decay mode is dominant and background neutrons cover and destroy the fundamental decay mode.
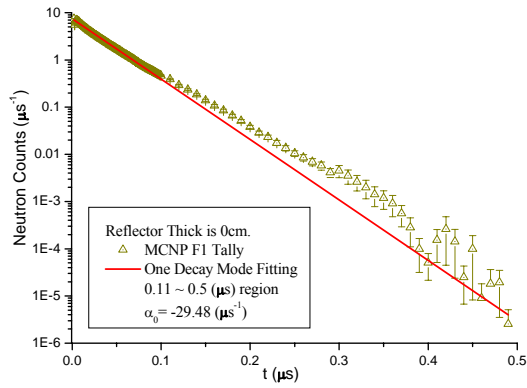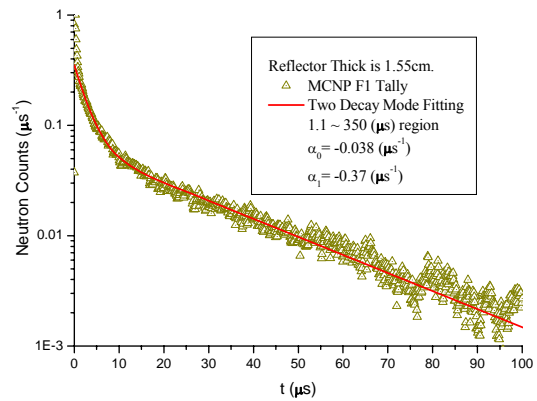
a
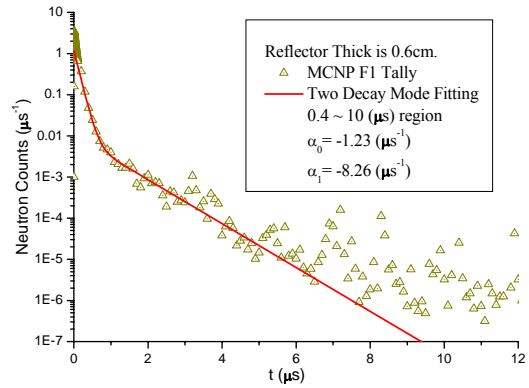
b
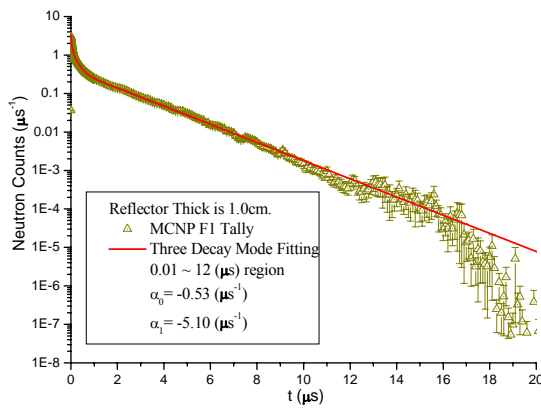
c

d

Fig. a. Leakage neutron time distribution and its fitting with 0cm-thick reflector.

Fig. b. Leakage neutron time distribution and its fitting with 0.6cm-thick reflector.

Fig. c. Leakage neutron time distribution and its fitting with 1.0cm-thick reflector.

Fig. d. Leakage neutron time distribution and its fitting with 1.55cm-thick reflector.

6. DISCUSSION

Comparing the results given in Table II, we can see that all 2 time constants of 2G2R model are close to MCNP time fitting method, the imaginary experiments. Two reasons contribute much to this. Firstly, two-group calculation is included in 2G2R model. Second, all coupling parameters from Spriggs model are determined by MCNP running.

At the same time, fundamental time constants calculated by Spriggs model results are close to MCNP time fitting for critical and bare system, and are not for two systems in the middle. Two middle systems' results are close to MCNP time fitting's secondary time constant. These two features can be explained that Spriggs model's time constant reflect the dominant time decay behavior which may not be the fundamental decay mode.

Though alpha static method (MCNP4C) 's results are close to MCNP time fitting, the convergence of k' in alpha static equation (1) or (2) is departure from unit with a few percent error for two middle systems, which strongly lowers the results of alpha static method.

According to the discussion above, 2G2R model provides a simple way to analyze multiple time decay modes quantitatively.

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APPENDEX Effective multiplication factor, k_{eff}

To calculate effective multiplication factor, k_{eff}, we resolve the set of equations below,

$$\begin{vmatrix} -B_c^2 D_{1c} + \frac{1}{k_{eff}}\left(1-\beta_{eff}\right)v\Sigma_{f_{1c}} - \Sigma_{r_{1c}} & \frac{1}{k_{eff}}\left(1-\beta_{eff}\right)v\Sigma_{f_{2c}} & B_r^2 D_{1r} f_{rc_{11}} & 0 \\ \Sigma_{s_{12c}} & -B_c^2 D_{2c} - \Sigma_{a_{2c}} & 0 & B_r^2 D_{2r} f_{rc_{22}} \\ B_c^2 D_{1c} f_{cr_{11}} & 0 & -B_r^2 D_{1r} - \Sigma_{r_{1r}} & 0 \\ 0 & B_c^2 D_{2c} f_{cr_{22}} & \Sigma_{s_{12r}} & -B_r^2 D_{2r} - \Sigma_{a_{2r}} \end{vmatrix} = 0 \quad \text{(A1)}$$

The effective multiplication factor, k_{eff} and k_c are,

$$k_{eff} = \frac{1}{(1-f_{11})}\left[\frac{v\Sigma_{f_{1c}}/\Sigma_{r_{1c}}}{1+\tau_1 B_c^2} + \frac{v\Sigma_{f_{2c}}/\Sigma_{a_{2c}}}{\left(1+\tau_2 B_c^2\right)\left(1-f_{22}\right)}\left(\frac{\Sigma_{s_{12c}}/\Sigma_{r_{1c}}}{1+\tau_1 B_c^2} + f_{12}\right)\right].$$

$$= \frac{1}{(1-f_{11})}\left[k_{1c} + \frac{k_{2c}\left(k_{s_{12c}}+f_{12}\right)}{(1-f_{22})}\right] \approx \frac{k_c + \Delta}{(1-f_{11})} \quad \text{(A2)}$$

$$k_c = k_{1c} + k_{2c} k_{s_{12c}} = \frac{v\Sigma_{f_{1c}}/\Sigma_{r_{1c}}}{1+\tau_1 B_c^2} + \frac{v\Sigma_{f_{2c}}/\Sigma_{a_{2c}}}{1+\tau_2 B_c^2}\frac{\Sigma_{s_{12c}}/\Sigma_{r_{1c}}}{1+\tau_1 B_c^2} \quad \text{(A3)}$$