Calculation of Fission Yield by Macroscopic-Microscopic Method Based on Selective Channel Scission Model

Masayuki OHTA and Shoji NAKAMURA
Japan Atomic Energy Agency
2-4 Shirakata-Shirane, Tokai-mura, Naka-gun, Ibaraki, 319-1195, Japan
E-mail: ohta.masayuki@jaea.go.jp

The mass-distributions of fission yields for neutron-induced fissions of U-235 were calculated by a macroscopic-microscopic method based on the selective channel scission model. The present result was compared with the previous result from the aspect of fission modes.

1. Introduction

The selective channel scission (SCS) model has been proposed and developed to calculate fission yields for any nuclei [1-3]. The SCS model deals with the fission process for each channel. The fission yield is obtained from the penetrability of the “channel-dependent” fission barrier. In previous analysis [3], mass-distributions of fission yields were calculated on simple assumptions about the channel-dependent fission barriers. This calculation method is applicable to wide range of fissionable nuclei without adjustable parameters. However, there were discrepancies between the calculated results and experimental data of fission yield in the mass regions of A = 85–95 and A = 140–150.

In this work, the channel-dependent fission potentials were calculated by a macroscopic-microscopic method based on the idea of SCS. The mass-distribution of fission yield was calculated for the neutron-induced fission of U-235.

2. SCS Model and Calculation of Fission Potential

The SCS model deals with the fission process for each channel. The fission yields are calculated from the penetrabilities of the “channel-dependent” fission barriers E_f.

The basic definition of nuclear shape is given by

$$R(\theta) = \lambda^{-1} R_0 \left(1 + \sum_{n=1}^{N} \alpha_n P_n(\cos\theta) \right), \tag{1}$$

where λ^{-1} is the volume conservation, R_0 is the radius of spherical nucleus, α_n is the deformation parameter and P_n is Legendre polynomial.

A macroscopic-microscopic method is commonly used for the calculation of fission potential.

The total potential energy E of a deformed nucleus is defined as the summation of the liquid-drop energy E_{LDM} as a macroscopic term and the shell correction energy E_{shell} as a microscopic term in this method.

$$E = E_{\mathrm{LDM}} + E_{\mathrm{shell}}. \tag{2}$$

The E_{LDM} is derived from the surface energy E_{S} and the Coulomb energy E_{C} of the deformed nucleus.

$$E_{\mathrm{LDM}} = E_{\mathrm{S}} + E_{\mathrm{C}}. \tag{3}$$

The channel-dependent fission potentials were calculated by a macroscopic-microscopic method based on the idea of SCS. The surface energy E_{S} in the macroscopic term was obtained from an equation whose form was proportional to the surface area S of the deformed nucleus [4].

$$E_{\mathrm{S}} = \gamma\, S,$$
$$\gamma = \frac{1}{4\pi r_0^2} a_2 \left[1 - \kappa \left(\frac{N-Z}{A}\right)^2\right]. \tag{4}$$

The Coulomb energy E_{C} also in the macroscopic term was obtained by the Monte-Carlo integral of the Coulomb energy between differential volumes which were taken at random all over the region of the deformed nucleus (see **Fig.1**). Mersenne Twister [5] was used as a random number generator.

$$\begin{aligned} E_{\mathrm{C}} &= \frac{1}{2}\iint \frac{1}{4\pi\,\varepsilon_0} \frac{\rho_i \rho_j}{r_{ij}}\, d^3 r_i\, d^3 r_j \\ &= \frac{Ze^2}{4\pi\,\varepsilon_0} \frac{1}{2\mathrm{N}^2} \sum_{i \neq j}^{\mathrm{N}}\sum \frac{\rho_i \rho_j}{\rho^2} \frac{1}{r_{ij}}. \end{aligned} \tag{5}$$

The shell energy E_{shell} in the microscopic term was calculated approximately as follows. The two fission fragments (FP1 and FP2) were assigned to the shape of the deformed nucleus for a channel (see **Fig. 2**). The whole shell energy was calculated from the sum of the shell energies of the two deformed fission fragments assigned to the deformed nucleus (E_{sh1} and E_{sh2}).

$$E_{\mathrm{shell}} = E_{\mathrm{sh1}} + E_{\mathrm{sh2}}. \tag{6}$$

A calculation code [6] was used for the calculation of shell energy for each fission fragment.

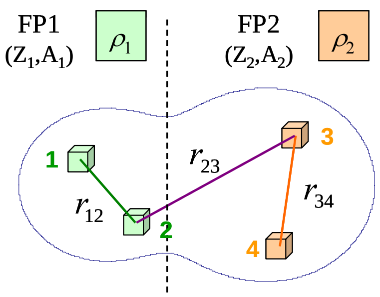


Fig. 1 Differential volumes taken in the deformed nucleus

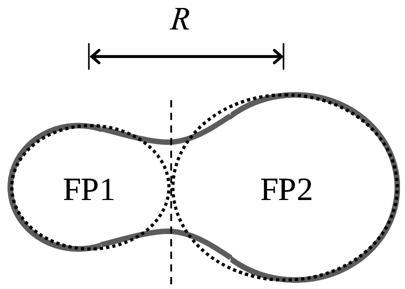


Fig. 2 Assignment of the two fragments to the deformed nucleus

The potential near the saddle point is approximated by the inverted parabola and the curvatures α is assumed as a constant for all humps, for simplicity. The tunnel probability P_i for the saddle point i is reduced as

$$P_i \approx \frac{1}{1+\exp\left[0.218\alpha\sqrt{\mu}\ \Delta E_i\right]}, \qquad (7)$$

in MeV and fm units, where μ = $A_1A_2/(A_1+A_2)$, $\Delta E_i = E_{fi} - E_x$ and A_1 and A_2 are the mass number of FP1 and FP2, respectively. In case of a two-humped potential, the probability P is deduced from tunnel probabilities for the two humps (P_A and P_B).

$$P = \frac{P_A\, P_B}{P_A + P_B}. \qquad (8)$$

The fission yields are obtained by summing up these probabilities all over fission channels.

3. Results and Discussions

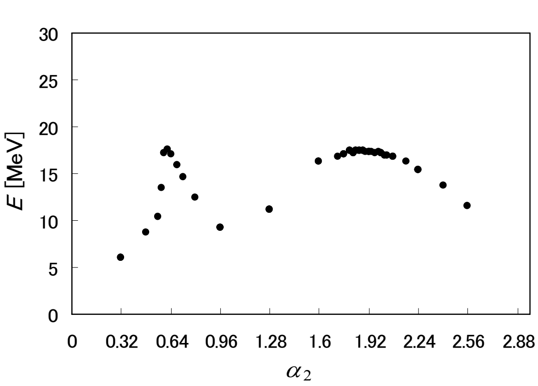
Figure 3-(a) shows an example of calculated fission potential for a channel. These potential calculations were carried out for about 230 channels that have high fission yields. The tunnel probability P was obtained for an excitation energy (e.g. E_x = 0), as shown in **Fig. 3-(b)**. The parameters α_2 at inner and outer saddle points were shown in **Figs. 3-(c)** and **3-(d)**, respectively.

Fission yields for the thermal neutron-induced fissions of U-235 were obtained as shown in **Fig. 3-(e)**. The α was taken as 0.2 in Eq. (7). Prompt neutron emission was not considered in the calculated fission yield. Meanwhile, the prompt neutron emission is considered for JENDL-3.3 data. It is known that the neutron multiplicity against mass number of fragment shows a saw-tooth curve [7]. Then, the calculated fission yield showed qualitative consistency with the data of JENDL-3.3. There were discrepancies in mass regions of A = 85–95 and A = 140–150 in previous analysis [3]. Although there were not such discrepancies in present result, fission yields were underestimated in mass regions above A = 150 and below A = 90.

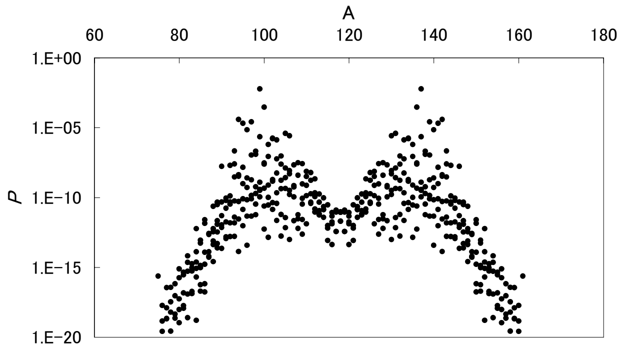
The shape elongation factor η was obtained at the saddle point deduced from JENDL-3.3 data in a previous analysis [2]. The η changed the trend at mass of fragments A ~ 130. The similar behavior appeared in the deformation parameter a_2 in Fig. 3-(c). The α_2 contributes significantly to the deformation of nucleus. It might depend on the existence of symmetric and asymmetric fissions.

4. Conclusions

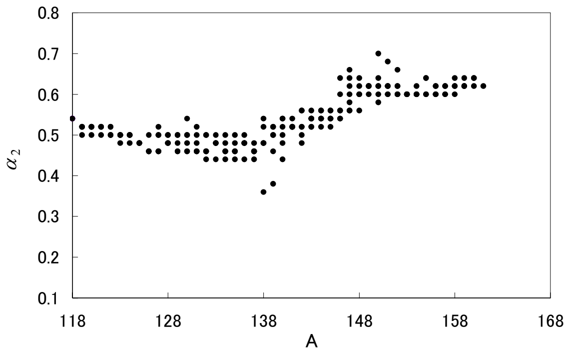
The channel-dependent fission potentials were calculated for the neutron-induced fission of U-235 by a macroscopic-microscopic method based on the selective channel scission model. The mass-distribution of fission yield was obtained for thermal neutron-induced fission of U-235.



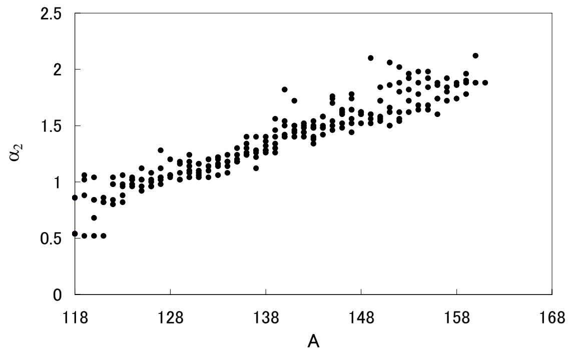
(a) Example of fission potential



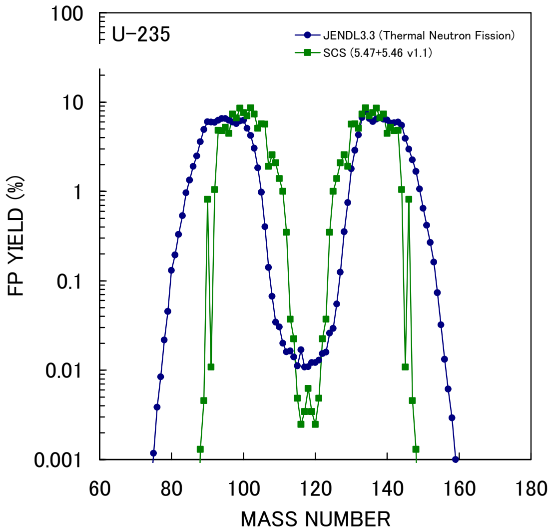
(b) Example of tunneling probability P



(c) Parameter α_2 at inner saddle point



(d) Parameter α_2 at outer saddle point



(e) Fission yield for thermal neutron

Fig. 3 Results for n+^{235}U

Acknowledgement

The computations were carried out with Fujitsu PRIMEPOWER HPC2500 at Japan Atomic Energy Agency (JAEA).

References

[1] A. Takahashi *et al*., *Jpn. J. Appl. Phys*., **40** (2001) 7031.

[2] M. Ohta *et al*., *Jpn. J. Appl. Phys*., **45** (2006) 6431.

[3] M. Ohta *et al*., *Proc. 2005 Symp. Nucl. Data*, JAEA-Conf 2006-009 (2006) p. 145.

[4] W.D. Myers and W.J. Swiatecki, *Nucl. Phys.*, **81** (1966) 1.

[5] M. Matsumoto and T. Nishimura, *ACM Trans. on Modeling and Computer Simulation*, **8** (1998) 3.

[6] F. Garcia *et al*., *Comp. Phys. Comm*., **120** (1999) 57.

[7] For example, *The New Encyclopaedia Britannica*, Encyclopaedia Britannica, Chicago, 1980 15th ed., Vol. 13, p. 301.